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Research Paper

CONVERGENCE ANALYSIS OF PROPORTIONAL-DERIVATIVE -TYPE ILC FOR LINEAR CONTINUOUS CONSTANT TIME DELAY SWITCHED SYSTEMS WITH OBSERVATION NOISE AND STATE UNCERTAINTIES

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Abstract. This article is concerned with the linear continuous time delay switching system with state uncertainties and observation noise. The goal of this study is to investigate how an internal switching mechanism and the efficacy of a conventional proportional-derivative ILC method is impacted by ambient noise for linear continuous-time switching systems. The findings demonstrate that learning gains and the dynamics of the subsystems, rather than the time-driven switching rule, are primarily responsible for the convergence and robustness of the control method. An appropriate selection of learning gains can ensure the control algorithm's convergence and resilience given any arbitrary time-varying switching rule.

Key Words: Iterative learning control, switched system, dynamical system, time delay, bounded state disturbance, bounded observation noise.
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1. Introduction

Theoretical research and practical applications of switched systems, which contain a given finite number of subsystems and switching signals, have recently received a great deal of interest. When a link in a network fails or is created, the connection topology frequently changes. Because the reference trajectory is established over a finite period, an ILC system repeatedly performs the same finite duration operation. The duration is referred to as the pass length, and each repetition is referred to as a pass. The system is brought back to its starting point when each pass is finished, so that the next pass may begin. The systems might diverge as a result of the states being reset, which could result in positioning problems. Through repeated completion of the same tasks, an ILC law that combines the knowledge from past passes with that from the current pass can eventually bring the output to the reference trajectory. Many ILC laws, including PID-type, P-type, and D-type ILC, PD-type, have been suggested for various kinds of systems. For instance, a hard disc drive's track following duty, a wafer manufacturing process's temperature management task, etc. When we refer to an extensive system, it means one that is made up of several subsystems that are connected by the system's extensive state vector, but each of which is managed based on its own input and output data. Examples of typical large scale systems include petrochemical operations, electricity systems, networked control systems, etc. According to Chen at. al. [1], a system for learning at the beginning that operates between two successive iterations, establishes the starting position at a certain location, and asymptotically converges is suggested.

In 1993, according to Hwangat. al. [2] the Derivative type ILC is built for reliable continuous-time systems, which are linear, and by this, we mean that the systems are fed a comparison of tracking error. One of the key issues that occurs with switched systems is stability, which has drawn the most attention. To analyze the stability of switched systems, a variety of techniques have been developed, and numerous helpful stability criteria have been defined in some articles. In order to ensure system stability and improve system performance, the dwell-time approach has been successful in determining the proper switching signals for switched systems that are subject to controlled switching signals

Ruan at. al. [4] offer a PID type control update method that follows non-repeated goal trajectories. The technique is demonstrated to be limited in the Lp norm sense. It is well knowledge that many engineering systems inevitably have time delays. It is possible for the system to become unstable if the time delays are not properly managed. A type of time delay system known as a neutral system depends not only on state delay but also on state derivative delay. In [3], the requirements for switching delay systems' delay-dependent exponential stability are provided [5]-[10]. Due to its hybrid nature, a switched system typically does not inherit subsystem characteristics [12].

In some cases, alternating between these reliable sub-systems may even cause the switched system to become unstable. For instance, the stability which is global exponential, trait of all subsystems cannot ensure the switched system has the same stability attribute. Therefore, switched systems are not immediately applicable to typical design and analysis techniques for systems without switching. Evidently, switched systems are rife with uncertainty, which complicates the research of switched systems even further. It is anticipated that adaptive control, which is an effective method for researching ambiguous non-switched systems, will also be useful for research of switched systems with uncertainty [13]-[16]. In reality, this presumption is frequently unfounded [9].

For a class of LCTSSs, which may be recognized by random time-driven switching signals and observation noise interference, the learning performance of a classic PD-type ILC scheme was examined by Xaun Yang et al. in 2018 [18]. A necessary condition of convergence and robustness is derived by incorporating using some lemma, and the impact of switching and noise is examined.

The rest of this essay is structured as follows. Preliminary, concept property and lemma related information are given in Section 2. The tracking effectiveness of a class of linear continuous time delay switched systems with observation noise and state uncertainties using PD- type ILC is examined in section 3. The paper is wrapped up in the final part.

2. **Basic and Mathematical Formulation**

Take into consideration a class of linear continuous time delay switching systems with state uncertainties:

$$\begin{cases} \dot{x}_k(t) & = A_{\sigma(t)}x_k(t) + D_{\sigma(t)}x_k(t-\tau) + B_{\sigma(t)}u_k(t) + \xi_k(t), \\ y_k(t) & = C_{\sigma(t)}x_k(t) + w_{\sigma,k}(t), \quad t \in \Omega = [0,T], \end{cases} \tag{2.1}$$

here

(1) $k \in \mathbb{N}$ represent the number of iterations, $\Omega = [0, T]$ denotes the time interval and $t \in \Omega$ denotes variable for time, τ denotes delay in time;
(2) $x_k(t)$ be element $\mathbb{R}^n$,, which stands for state vector, $u_k(t)$ is an element in $\in \mathbb{R}^m$, which stands for input vector and $y_k(t)$ is element in $\mathbb{R}^l$ is output vector.
(3) $w_{\sigma,k}(t) \in \mathbb{R}^l$ and $\xi_k(t)$ denotes the bounded state disturbance bounded observation noise with $\|w_{\sigma(i),k}\|_p \leq w_{i,0}$ and $\|\xi_k(t)\|_p \leq b_\xi$;
(4) $A_\sigma(t)$ be the matrix in $\in \mathbb{R}^{n\times n}$, $B_{\sigma(t)}$ be the matrix in $\mathbb{R}^{n\times m}$ and $C_{\sigma(t)}$ are also matrix in $\mathbb{R}^{l\times n}$, this all type of matirx are known as system matrices;
(5) $\sigma : [0, T] \to Q, \quad Q = \{1, 2, \cdots, q\}$ over a period of time, $[0, T]$ denotes a piecewise constant function which is known as the switching rule.

Without harming generality, it is thought to be characterized as

$$\sigma(t) = i = \begin{cases} 1, & t \text{ belong to } [0, t_1), \\ 2, & t \text{ belong to } [t_1, t_2), \\ \vdots & \\ q, & t \text{ belong to } [t_{q-1}, T]. \end{cases} \tag{2.2}$$

Therefore, the matrices group $(A_{\sigma(t)}, B_{\sigma(t)}, C_{\sigma(t)}, D_{\sigma(t)})$, for $\sigma(t)$ belong to $Q = \{1, 2, \cdots, q\}$ are a component of the ensuing the following set

$$\{(A_1, B_1, C_1, D_1), (A_2, B_2, C_2, D_2), \cdots, (A_q, B_q, C_q, D_q)\}$$

Satisfied (2.2), the system (2.1) is perhaps reformed as

$$\begin{cases} \dot{x}_k(t) &= A_i x_k(t) + B_i u_k(t) + D_i x_k(t-\tau) + \xi_k(t), \\ y_k(t) &= C_i x_k(t) + w_{i,k}(t), \quad t \text{ belong to } \Omega = [0, T], i \in Q. \end{cases} \tag{2.3}$$

Keep in mind that the dynamic system (2.3) can function repeatedly across the range $[0, T]$ of time, which is finite, even if the precise dynamics may not be known.

Consider the scheme, which is known as PD- type ILC as follows:

$$u_{k+1}(t) = u_k(t) + \Gamma_{p,i} e_k(t) + \Gamma_{d,i} \dot{e}_k(t), \quad i \in Q = \{1, 2, \cdot, q\}, k = 1, 2, 3, ... \tag{2.4}$$

is imposed the k^{th} term of error, which is denoted by $e_k(t)$ and define as $e_k(t) = y_d(t) - y_k(t)$, for any t belong to finite time interval $[0, T]$ is known as the tracking error, and $\Gamma_{d,i} \in \mathbb{R}^{m\times l}$ and $\Gamma_{p,i} \in \mathbb{R}^{m\times l}$ are known as derivative gains and proportional gains, respectively. The purpose is that the output of the system (2.3) asymptotically converges to the given targeted or reference trajectory, which is indicated by $y_d(t)$, in time period $t \in [0, T]$ as exactly as feasible or when the iteration number tence to infinity follows into the vicinity of $y_d(t)$, that is,

$$\lim_{k\to\infty} \|e_{k+1}(\cdot)\|_p = 0 \text{ or } \lim_{k\to\infty} \sup\|e_{k+1}(\cdot)\|_p \leq \eta.$$

To object of this problem, to find the sequence $\{u_k(t) : k \in \mathbb{Z}_+\}$ such that $\{y_k(t)\}$ tends to $y_d(t)$ for (2.1) with PD-type ILC scheme(2.2).

Definition 2.1. [18] Consider the vector valued function $g : I \subseteq \mathbb{R}^+ \to \mathbb{R}^n$ defined by

$$g(t) = [g_1(t), g_2(t), \cdots, g_n(t)]^T,$$

its Lebesgue -p norm is defined as

$$\|g(\cdot)\|_p = \left[\int_I \left(\max_{1\leq j\leq n}\{|g_j(t)|\}\right)^p dy\right]^{\frac{1}{p}}, 1 \leq p \leq \infty.$$

Definition 2.2. [19]

For a given vector valued function $f(t) \in \mathbb{R}^n, g(t) \in \mathbb{R}^n$, the convolution integral is described as

$$(f * g)(t) = \int_I f(t-s)g(s)ds.$$

From definition (2.1) and (2.2), The convolution integrals generalized Young inequality (GYI) is stated as

$$\|f(\cdot)\|_q \|g(\cdot)\|_p \geq \|(f * g)(\cdot)\|_r, \tag{2.5}$$

for all $1 \leq p, q, r < \infty$ satisfying

$$1/r = 1/p + 1/q.$$

In particular, if p and r are equal, then inequality (2.5) , we get

$$\|f\|_1 \|g\|_p \geq \|f * g\|_p, \tag{2.6}$$

when $p = r$.

The following are the system's (2.3) basic presumptions:

Assumption 1: Every operation begins at the same starting place. In this paper, it's thought to be so $y_d(0) = y_k(0)$, for all $k = 1, 2, \cdots$.

Assumption 2: The given targeted or reference or desired output $y_d(t)$ is invariant in the process of iteration over a time interval $[0, T]$.

Assumption 3: The switched sequence $\sigma(t)$ still maintains iteration invariance and at the first iteration, it is randomly chosen.

Assumption 4: For every $t \in [0, T]$, there is $\Delta x_k(-t) = 0$.

Assumption 5: Over every time sub-interval $[t_{i-1}, t_i], i \in Q,$, the observation noise is arbitrarily constrained, it means, $w_{i,k}(t) \leq w_{i,0}$ where, each time subinterval's value of $w_{i,0}$ is to little enough non-negative constant.

Assumption 6: The state uncertainty (disturbance) is bounded, that is, $\|\xi_k\|_p \leq b_\xi$.

Assumption 7: Regarding the specified intended result $y_d(t)$, the only thing present is a desired control input $u_d(t)$ and a desired $x_d(t)$ s. t.

$$\begin{cases} \dot{x}_d(t) & = A_i x_d(t) + D_i x_d(t-\tau) + B_i u_d(t) + \xi_d(t), \\ y_d(t) & = C_i x_d(t), \quad t \in \Omega = [0, T], i \in Q. \end{cases}$$

Here τ denotes the time delay so that the dwell times of every subsystem exceed the delay times. That is,

$$\tau < t_i - t_{i-1}, \quad \forall i \in Q.$$

3. Main Results

Lemma 3.1. [19] *Assume that $\{b_k\}$ is a positive sequence of a real sequence defined as follows:*

$$b_k \leq c_1 b_{k-1} + c_2 b_{k-2} + \cdots + c_n b_{k-n} + \varepsilon_k, \quad k = n+1, n+2, \cdots,$$

with the starting value b_l for every $l = 1, 2, \cdots, n$, where $\{\varepsilon_k\}$ is another specified real sequence. If the coefficient satisfy $c_j \geq 0$ and

$$c = \sum_{j=1}^{n} c_j < 1,$$

then the $\limsup_{k\to\infty} \varepsilon_k \le \varepsilon$ *implies that*

$$\limsup_{k\to\infty} b_k \le \frac{\varepsilon}{1-c}.$$

In particular, $\lim_{k\to\infty} b_k = 0$, *provided that* $\varepsilon = 0$.

Theorem 3.2. *Consider the scheme (2.4) that is imposed on the system (2.3), which is defined by the switching rule (2.4) and is affected by uncertainties and noise. Assume that the system (2.3) satisfies assumes from A1 to A7. If the* A_i, B_i, C_i *and* D_i *are system dynamics together with the learning gains* $\Gamma_{d,i}$ *and* $\Gamma_{p,i}$ *satisfy*

$$\|C_i \exp(A_i \cdot (\cdot))(A_i B_i \Gamma_{d,i} + B_i \Gamma_{p,i})\|_1 + \|I - C_i B_i \Gamma_{d,i}\|_\infty = \rho_i < 1, \tag{3.1}$$

for every sub-system, then the system output $y_k(t)$ *can approach the neighborhood of the targeted trajectory* $y_d(t)$ *in the whole time interval, as the iteration num tends to infinity.*

Proof. Firstly, consider the input control signal $u_k(t)$ in the k^{th} trial over time sub interval $[t_{i-1}, t_i](i \in Q)$, the state response trajectory of the system (2.3) is formally represented as

$$\begin{aligned} x_{k+1}(t) = {} & \exp(A_i \cdot (t - t_{i-1}))x_{k+1}(t_{i-1}) \\ & + \int_{t_{i-1}}^{t} \exp(A_i \cdot (t-s))D_i x_{k+1}(s-\tau)(s)ds \\ & + \int_{t_{i-1}}^{t} \exp(A_i \cdot (t-s))B_i u_{k+1}(s)ds \\ & + \int_{t_{i-1}}^{t} \exp(A_i \cdot (t-s))\xi_{k+1}(s)ds. \end{aligned}$$

Using the recursive relationship of tracking errors, the tracking error $e_{k+1}(t)$ is therefore described as follows:

$$\begin{aligned}
e_{k+1}(t) &= y_d(t) - y_{k+1}(t) \\
&= y_d(t) - y_k(t) - [y_{k+1}(t) - y_k(t)] \\
&= e_k(t) - C_i \exp(A_i \cdot (t - t_{i-1}))(x_{k+1}(t_{i-1}) - x_k(t_{i-1})) \\
&- C_i \int_{t_i}^{t} \exp(A_i \cdot (t-s)) D_i [x_{k+1}(s-\tau) - x_k(s-\tau) ds \\
&- C_i \int_{t_{i-1}}^{t} \exp(A_i \cdot (t-s)) B_i [u_{k+1}(s) - u_k(s)] ds \\
&- C_i \int_{t_{i-1}}^{t} \exp(A_i \cdot (t-s)) [\xi_{k+1}(s) - \xi_k(s) ds - (w_{i,k+1}(t) - w_{i,k}).
\end{aligned} \tag{3.2}$$

Now, we consider the PD type ILC as an updating law (2.4), which is a substitute in the above equation (3.2), we can easily calculate as follows:

$$\begin{aligned}
e_{k+1}(t) &= e_k(t) - C_i \exp(A_i \cdot (t - t_{i-1}))(x_{k+1}(t_{i-1}) - x_k(t_{i-1})) \\
&- C_i \int_{t_i}^{t} \exp(A_i \cdot (t-s)) D_i [x_{k+1}(s-\tau) - x_k(s-\tau) ds \\
&- C_i \int_{t_{i-1}}^{t} \exp(A_i \cdot (t-s)) B_i [\Gamma_{p,i} e_k(s) + \Gamma_{d,i} \dot{e}_k(s)] ds \\
&- C_i \int_{t_{i-1}}^{t} \exp(A_i \cdot (t-s)) [\xi_{k+1}(s) - \xi_k(s) ds - (w_{i,k+1}(t) - w_{i,k}) \\
&= e_k(t) - C_i \exp(A_i \cdot (t - t_{i-1}))(x_{k+1}(t_{i-1}) - x_k(t_{i-1})) \\
&- C_i \int_{t_i}^{t} \exp(A_i \cdot (t-s)) D_i [x_{k+1}(s-\tau) - x_k(s-\tau) ds \\
&- C_i \int_{t_{i-1}}^{t} \exp(A_i \cdot (t-s)) B_i \Gamma_{p,i} e_k(s) ds \\
&- C_i \int_{t_{i-1}}^{t} \exp(A_i \cdot (t-s)) [\xi_{k+1}(s) - \xi_k(s) ds \\
&- C_i \int_{t_{i-1}}^{t} \exp(A_i \cdot (t-s)) B_i \Gamma_{d,i} \dot{e}_k(s) ds - (w_{i,k+1}(t) - w_{i,k}).
\end{aligned} \tag{3.3}$$

By using the partial integration approach, it is possible to rearrange the last term in equation (3.3) to become as follows:

$$
\begin{aligned}
C_i \int_{t_{i-1}}^{t} &\exp(A_i\cdot(t-s))B_i\Gamma_{d,i}\dot{e}_k(s)ds \\
&= C_i \exp(A_i\cdot(t-s))B_i\Gamma_{d,i}e_k(s)\big|_{s=t_{i-1}}^{s=t} \\
&- C_i \int_{t_{i-1}}^{t} \exp(A_i\cdot(t-s))(A_iB_i\Gamma_{d,i} \\
&+ B_i\Gamma_{p,i})e_k(s)ds.
\end{aligned}
\tag{3.4}
$$

Substituing (3.4) into (3.3) yields

$$
\begin{aligned}
e_{k+1}(t) = e_k(t) &- C_i\exp(A_i\cdot(t-t_{i-1}))(x_{k+1}(t_{i-1}) - x_k(t_{i-1})) \\
&- C_i\int_{t_i}^{t}\exp(A_i\cdot(t-s))D_i[x_{k+1}(s-\tau) - x_k(s-\tau)ds \\
&- C_i\int_{t_{i-1}}^{t}\exp(A_i\cdot(t-s))B_i\Gamma_{p,i}e_k(s)ds \\
&- C_i\int_{t_{i-1}}^{t}\exp(A_i\cdot(t-s))[\xi_{k+1}(s) - \xi_k(s)ds \\
&- C_i\exp(A_i\cdot(t-s))B_i\Gamma_{d,i}e_k(s)\big|_{s=t_{i-1}}^{s=t} \\
&- C_i\int_{t_{i-1}}^{t}\exp(A_i\cdot(t-s))(A_iB_i\Gamma_{d,i} + B_i\Gamma_{p,i})e_k(s)ds \\
&- (w_{i,k+1}(t) - w_{i,k}).
\end{aligned}
\tag{3.5}
$$

Step 1: If t belongs to the first sub-interval $t \in \Omega_1$. The first subsystem is turned on in this situation. Taking $t_0 = 0$, the tracking error's recursive connection (3.4) becomes

$$\begin{aligned} e_{k+1}(t) = e_k(t) &- C_i \exp(A_1 \cdot (t))(x_{k+1}(0) - x_k(0)) \\ &- C_1 \int_0^t \exp(A_1 \cdot (t-s)) D_1 [x_{k+1}(s-\tau) - x_k(s-\tau)] ds \\ &- C_1 \int_0^t \exp(A_1 \cdot (t-s)) B_1 \Gamma_{p,i} e_k(s) ds \\ &- C_1 \int_0^t \exp(A_1 \cdot (t-s)) [\xi_{k+1}(s) - \xi_k(s)] ds \\ &- C_1 \exp(A_1 \cdot (t-s)) B_1 \Gamma_{d,1} e_k(s) \Big|_{s=t_{i-1}}^{s=t} \\ &- C_1 \int_0^t \exp(A_1 \cdot (t-s)) (A_1 B_1 \Gamma_{d,1} \\ &+ B_1 \Gamma_{p,1}) e_k(s) ds. - (w_{1,k+1}(t) - w_{1,k}). \end{aligned} \tag{3.6}$$

Using first assumption A1, which is $(x_{k+1}(0) - x_k(0)) = 0$. Thus, we have

$$\begin{aligned} e_{k+1}(t) = (I - C_1 B_1 \Gamma_{d,1}) e_k(t) &- C_1 \int_0^t \exp(A_1 \cdot (t-s)) (A_1 B_1 \Gamma_{d,1} + B_1 \Gamma_{p,1}) e_k(s) ds \\ &- C_1 \int_0^t \exp(A_1 \cdot (t-s)) D_1 [x_{k+1}(s-\tau) - x_k(s-\tau)] ds \\ &- C_1 \int_0^t \exp(A_1 \cdot (t-s)) [\xi_{k+1}(s) - \xi_k(s)] ds \\ &- (w_{1,k+1}(t) - w_{1,k}). \end{aligned} \tag{3.7}$$

Since $\Delta w_{1,k}(t) = w_{1,k+1}(t) - w_{1,k}$, then we get as follows:

$$\begin{aligned} e_{k+1}(t) = (I - C_1 B_1 \Gamma_{d,i}) e_k(t) &- C_1 \int_0^t \exp(A_1 (t-s)) (A_1 \cdot B_1 \Gamma_{d,1} \\ &+ B_1 \Gamma_{p,1}) e_k(s) ds - C_1 \int_0^t \exp(A_1 \cdot (t-s)) D_1 \Delta x_k(s-\tau) ds \\ &- C_1 \int_{t_0}^t \exp(A_1 \cdot (t-s)) \Delta \xi_k(s) ds - \Delta w_{1,k}(t). \end{aligned} \tag{3.8}$$

where $\Delta x_k(s-\tau) = x_{k+1}(s-\tau) - x_k(s-\tau)$, $\Delta \xi_k(s) = \xi_{k+1}(s) - \xi_k(s)$ and $\Delta w_{1,k} = w_{1,k+1}(t) - w_{1,k}(t)$. Firstly, applying the Lebesgue-p norm on two sides of the equation (3.8) and using the definitions (2.1) and (2.2), we get

$$
\begin{aligned}
\|e_{k+1}(\cdot)\|_p \leq (\|(I - C_1 B_1 \Gamma_{d,1})\|_\infty + &\|C_1 \exp(A_1(t-s))(A_1 \cdot B_1 \Gamma_{d,i} \\
&+ B_1 \Gamma_{p,1})\|_1) e_k(\cdot)\|_p + \|C_1 \exp(A_1 \cdot (t-s)) D_1\|_p \|\Delta x_k(s-\tau)\|_p \\
&+ \|\exp(A_1(t-s)\|_p \|\Delta \xi_k(t)\|_p + \|\Delta w_{1,k}(t)\|_p \\
&\leq \|(I - C_1 B_1 \Gamma_{d,1})\|_\infty \|e_k(\cdot)\|_p + \|C_1 \exp(A_1 \cdot (t-s)) \\
&(A_1 B_1 \Gamma_{d,1} + B_1 \Gamma_{p,1})\|_1 \|e_k(\cdot)\|_p \\
&+ \|C_1 \exp(A_1(t-s)) D_1\|_p \gamma_0 + \|C_1 \exp(A_1 \cdot (t-s)\|_p b_\xi + 2 b_{w_{1,0}}
\end{aligned}
\tag{3.9}
$$

Where $\|\Delta x_k(s-\tau)\|_p \leq \gamma_0$, and we can observe that

$$
\begin{aligned}
\|\Delta \xi_k(t)\|_p &\leq \|\xi_{k+1}(t)\|_p + \|\xi_k(t)\|_p \leq 2 b_\xi, \\
\|\Delta w_{1,k}(t)\|_p &\leq \|w_{1,k+1}(t)\|_p + \|w_{1,k}(t)\|_p \leq 2 w_{1,0},
\end{aligned}
$$

So, $\|\Delta \xi_k(t)\|_p$ and $\|\Delta w_{1,k}(t)\|_p$ are bounded by $2b_\xi$ and $2w_{1,0}$ respectively. Taking the supremum of the equation (3.9) with the assumption $\rho_1 < 1$ and applying the Lemma 3.1, we conclude that

$$
\begin{aligned}
\lim_{k\to\infty} \|e_{k+1}(\cdot)\|_p \leq &\frac{\|C_1 \exp(A_1 \cdot (t-s)) D_1\|_p \gamma_0}{1-\rho_1} \\
&\frac{+\|C_1 \exp(A_1 \cdot (t-s)\|_p b_\xi + 2 b_{w_{1,0}}}{1-\rho_1} = \frac{\delta_1}{1-\rho_1},
\end{aligned}
\tag{3.10}
$$

over $[0, t_1)$, where $\delta_1 = \|C_1 \exp(A_1 \cdot (t-s)) D_1\|_p \gamma_0 + \|C_1 \exp(A_1 \cdot (t-s)\|_p b_\xi + 2 b_{w_{1,0}}$. In other words, the 1^{st} sub-system's output can follow the targeted trajectory towards a neighborhood on $\Omega_1 = [0, t_1)$.

Step 2: In the second step, t belongs to the second sub-system, $[t_1, t_2)$, The second subsystem is switched on in this situation. The tracking error (3.6) is expressed as follows:

$$
\begin{aligned}
e_{k+1}(t) = {} & e_k(t) - C_2 \exp(A_2 \cdot (t-t_1))(x_{k+1}(t_1) - x_k(t_1)) \\
& - C_2 \int_{t_1}^{t} \exp(A_2 \cdot (t-s)) D_2 [x_{k+1}(s-\tau) - x_k(s-\tau)] ds \\
& - C_2 \int_{t_1}^{t} \exp(A_2 \cdot (t-s)) [\xi_{k+1}(s) - \xi_k(s)] ds \\
& - C_2 \exp(A_2 \cdot (t-s)) B_2 \Gamma_{d,2} e_k(s) \big|_{s=t_1}^{s=t} \\
& - C_2 \int_{t_1}^{t} \exp(A_2 \cdot (t-s)) (A_2 B_2 \Gamma_{d,2} + B_2 \Gamma_{p,2}) e_k(s) ds \\
& - (w_{2,k+1}(t) - w_{2,k}) \\
= {} & (I - C_2 B_2 \Gamma_{d,i}) e_k(t) \\
& - C_2 \int_{t_1}^{t} \exp(A_2 \cdot (t-s)) (A_2 B_2 \Gamma_{d,2} + B_2 \Gamma_{p,2}) e_k(s) ds \\
& - C_2 \exp(A_2 \cdot (t-t_1)) \Delta x_k(t_1) + C_2 \exp(A_2 \cdot (t-t_1)) B_2 \Gamma_{d,2} e_k(t_1) \\
& - C_2 \int_{t_1}^{t} \exp(A_2 \cdot (t-s)) D_2 \Delta x_k(s-\tau) ds \\
& - C_2 \int_{t_1}^{t} \exp(A_2 \cdot (t-s)) \Delta \xi_k(s) ds + \Delta w_{2,k}(t).
\end{aligned}
\tag{3.11}
$$

Where $\Delta x(t_1)$ is equal to $x_{k+1}(t_1) - x_k(t_1)$, $\Delta x_k(s-\tau) = x_{k+1}(s-\tau) - x_k(s-\tau)$, $\Delta \xi_k(s) = \xi_{k+1}(s) - \xi_k(s)$ and $\Delta w_{2,k} = w_{2,k+1}(t) - w_{2,k}(t)$.

Using the generalized Young inequality of the convolution integral and the taking Lebesgue -p norm on both sides of the equation (3.11), and applying the definition (2.1) and (2.2), we can formulate as

$$\begin{aligned}
&\|e_{k+1}(\cdot)\|_p \\
&\le (\|I - C_2B_2\Gamma_{d,2}\|_\infty + \|C_2\exp(A_2\cdot(\cdot))(A_2B_2\Gamma_{d,2} + B_2\Gamma_{p,2})\|_1)\|e_k(\cdot)\|_p \\
&+\|C_2\exp(A_2\cdot(t-t_1))\|_p\|\Delta x_k(t_1)\|_p + \|C_2\exp(A_2\cdot(\cdot))D_2\|_p\|\Delta x_k(s-\tau)\|_p \\
&+\|C_2\exp(A_2\cdot(\cdot))\|_p\|\Delta\xi_k(t)\|_p + \|C_2\exp(A_2\cdot(\cdot))B_2\Gamma_{d,2}\|_p\|e_k(t_1)\|_p \\
&\qquad +\|\Delta w_{2,k}(t)\|_p
\end{aligned}$$

$$\begin{aligned}
&\le (\|(I - C_2B_2\Gamma_{d,2})\|_\infty + \|C_2\exp(A_2\cdot(\cdot))(A_2B_2\Gamma_{d,2} + B_2\Gamma_{p,2})\|_1)\|e_k(\cdot)\|_p \\
&+\|C_2\exp(A_2\cdot(t-t_1))\|_p\|\Delta x_k(t_1)\|_p + \|C_2\exp(A_2\cdot(\cdot))D_2\|_p\gamma_1 \\
&+\|C_2\exp(A_2\cdot(\cdot))B_2\Gamma_{d,2}\|_p\|e_k(t_1)\|_p + \|C_2\exp(A_2\cdot(\cdot))\|_p b_\xi + 2b_{w_{2,0}} \\
&= \rho_2\|e_k(\cdot)\|_p + \|C_2\exp(A_2\cdot(t-t_1))\|_p\|\Delta x_k(t_1)\|_p \\
&\qquad +\|C_2\exp(A_2\cdot(t-s))D_2\|_p\gamma_1 \\
&+\|C_2\exp(A_2\cdot(\cdot))B_2\Gamma_{d,2}\|_p\|e_k(t_1)\|_p + \|C_2\exp(A_2\cdot(t-s))\|_p b_\xi + 2b_{w_{2,0}}.
\end{aligned} \tag{3.12}$$

where $\|\Delta x_k(s-\tau)\|_p \le \gamma_1, \|\Delta\xi_k\|_p \le 2b_\xi, \|\Delta w_{2,k}\|_p \le 2b_{w_{2,0}}$. It is seen that the proving procedure starts with the first sub-interval Ω_1 that

$$\begin{aligned}
&\lim_{k\to\infty}\|e_{k+1}(\cdot)\|_p \\
&\le \frac{\|C_1\exp(A_1\cdot(t-s))D_1\|_p\gamma_0 + \|C_1\exp(A_1\cdot(t-s)\|_p b_\xi + 2b_{w_{1,0}}}{1-\rho_1} \\
&= \frac{\delta_1}{1-\rho_1},
\end{aligned}$$

Where $\delta_1 = \|C_1\exp(A_1\cdot(t-s))D_1\|_p\gamma_0 + \|C_1\exp(A_1\cdot(t-s)\|_p b_\xi + 2b_{w_{1,0}}$ satiesfy on first subinterval Ω_1, which implies both $\lim_{k\to\infty}\sup\|\Delta x_k(t_1)\|_p < \infty$ and $\lim_{k\to\infty}\sup\|e_k(t_1)\|_p < \infty$ are satiesfied. Now indecating $\lim_{k\to\infty}\sup\|\Delta x_k(t_1)\|_p = \alpha_1$ and $\lim_{k\to\infty}\sup\|e_k(t_1)\|_p = \beta_1$, the inequality (3.12) can be written as follows:

$$\begin{aligned}
\lim_{k\to\infty}\sup\|e_{k+1}(\cdot)\|_p \le \rho_2\|e_k(\cdot)\|_p &+ \|C_2\exp(A_2\cdot(\cdot))\|_p\alpha_1 \\
&+\|C_2\exp(A_2\cdot(\cdot))D_2\|_p\gamma_1 \\
&+\|C_2\exp(A_2\cdot(\cdot))B_2\Gamma_{d,2}\|_p\beta_1 \\
&+\|C_2\exp(A_2\cdot(\cdot))\|_p b_\xi + 2b_{w_{2,0}}.
\end{aligned} \tag{3.13}$$

Again, applying Lemma 3.1, it follows that

$$\lim_{k\to\infty}\sup\|e_{k+1}(\cdot)\|_p \le \frac{1}{\rho_2}\|C_2\exp(A_2\cdot(\cdot))\|_p\alpha_1+\|C_2\exp(A_2\cdot(\cdot))D_2\|_p\gamma_1 +\|C_2\exp(A_2\cdot(\cdot))B_2\Gamma_{d,2}\|_p\beta_1+\|C_2\exp(A_2\cdot(\cdot))\|_p b_\xi + 2b_{w_{2,0}}$$

$$= \frac{\delta_2}{1-\rho_2}. \tag{3.14}$$

where $\delta_2 = \|C_2\exp(A_2\cdot(\cdot))\|_p\alpha_1+\|C_2\exp(A_2\cdot(\cdot))D_2\|_p\gamma_1+\|C_2\exp(A_2\cdot(\cdot))B_2\Gamma_{d,2}\|_p\beta_1 +\|C_2\exp(A_2\cdot(\cdot))\|_p b_\xi + 2b_{w_{2,0}}$. Comparably repeating the aforementioned proof procedure for t 1 belong to i^{th} sub-interval, that is $t \in \Omega_i, (i = 1, 2, \cdot, q)$ and indicating $\lim_{k\to\infty}\sup\|\Delta x_k(t_{i-1})\|_p = \alpha_{i-1}$ and $\lim_{k\to\infty}\sup\|e_k(t_{i-1})\|_p = \beta_{i-1}$, In light of the inequalities, we may say

$$\lim_{k\to\infty}\sup\|e_{k+1}(\cdot)\|_p \le \frac{\|C_i\exp(A_i\cdot(\cdot))\|_p\alpha_{i-1}+\|C_i\exp(A_i\cdot(\cdot))D_i\|_p\gamma_{i-1}}{1-\rho_i} + \frac{\|C_i\exp(A_i\cdot(\cdot))B_i\Gamma_{d,i}\|_p\beta_{i-1}+\|C_i\exp(A_i\cdot(\cdot))\|_p b_\xi + 2b_{w_{i,0}}}{1-\rho_i}$$

$$= \frac{\delta_i}{1-\rho_i}, \tag{3.15}$$

satisfied on the time sub-interval $\Omega_i, (i = 1, 2, \cdots, q)$, where $\delta_i = \|C_i\exp(A_i\cdot(\cdot))\|_p\alpha_{i-1}+\|C_i\exp(A_i\cdot(\cdot))D_i\|_p\gamma_{i-1} +\|C_i\exp(A_i\cdot(\cdot))B_i\Gamma_{d,i}\|_p\beta_{i-1}+\|C_i\exp(A_i\cdot(\cdot))\|_p b_\xi + 2b_{w_{i,0}}$. In other words, throughout successive time intervals from Ω_1 to Ω_q, the output can converge into a neighborhood of the targeted or reference or desired output trajectory $y_d(t)$, and it also does for the whole time period Ω. This proof is complete. □

Remark 3.3. If $x_k(t-\tau) = 0$ and $\xi_k(0) = 0, \forall k \in \mathbb{N}$, then result become same as in [18].

4. Conclusion

The impact of traditional PD-type ILC on the LCTDSS with state uncertainties and observation noise has been examined in this study. The findings demonstrate that the control method is convergent, despite the fact that switching may take place at any instant when noise is present, and resilience may be ensured in the presence of bounded noise. We examine the impact of environmental noise and state time delay on tracking performance. There is also the option to analyze different ILC types for systems with many inputs and outputs that have a nonlinear continuous time delay.

5. Conflicts of interest

There are no conflicts to declare.

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References

[1] Chen, Y.Q., Wen, C.Y., Gong, Z., and Sun, M.X., *An Iterative Learning Controller with Initial State Learning*, IEEE Transactions on Automatic Control, **44** (1999),371376.

[2] Hwang, Dong-Hwan, Kim, Byung Kook, and Bien, Zeungnam, *Decentralized iterative learning control methods for large scale linear dynamic systems*, International Journal of Systems Science, **24(12)** (1993), 22392254.

[3] X. Sun, J. Zhao, D.J. Hill, *Stability and L2-gain analysis for switched systems: a delay-dependent method*, Automatica , **42(10)** (2006),17691774

[4] Ruan, X. E., Bien, Z. Z., and Park, K. H. *Decentralized iterative learning control to large-scale industrial processes for nonrepetitive trajectories tracking*, IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans, **38(1)** (2008), 238252

[5] D. Du, B. Jiang, P. Shi, S. Zhou, *H_∞ filtering of discrete-time switched systems with state delays via switched Lyapunov function approach*, IEEE Trans. Autom. Control, **52(8)** (2007), 15201525

[6] D. Du, B. Jiang, P. Shi, S. Zhou, *Robust $l_2 - l_\infty$ control for uncertain discrete-time switched systems with delays*, Circuits Syst. Signal Process **25(6)** (2006), 729744 (2006)

[7] D. Du, B. Jiang, S. Zhou,*Delay-dependent robust stabilization of uncertain discrete-time switched systems with time-varying state delay*, Int. J. Syst. Sci., **39(3)** (2008), No. 305313

[8] J. Liu, X. Liu, W. Xie, *Exponential stability of switched stochastic delay systems with non-linear uncertainties*, Int. J. Syst. Sci. **40(6)** (2009), 637648

[9] S.K. Nguang, P. Shi, *Fuzzy H_∞ output feedback control of nonlinear systems under sampled mea- surements*, Automatica, **39(12),** (2003), 21692174

[10] S.K. Nguang, P. Shim, *H_∞ fuzzy output feedback control design for nonlinear systems: an LMI ap- proach*, IEEE Trans. Fuzzy Syst., **11(3)** (2003), 331340

[11] Wu, C., Zhao, J., and Sun, X. M., *Adaptive tracking control for uncertain switched systems under asynchronous switching*, International Journal of Robust and Nonlinear Control, **25(17)** (2015), 3457-3477

[12] Su YF, Huang J., *Cooperative output regulation with application to multi-agent consensus under switching network*, IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics, **42(3)** (2012), No. 1-2, 864875

[13] Bernardo Md, Montanaro U, Santini S., *Hybrid model reference adaptive control of piecewise affine systems*, IEEE Transactions on Automatic Control **58(2)** (2013), 304315

[14] Lin Z, Saberi A, Stoorvogel AA, *An improvement to the low gain design for discrete-time linear systems in the presence of actuator saturation nonlinearity*, International Journal of Robust and Nonlinear Control **10(3)** (2000), No. 1-2, 117135

[15] Narendra KS, Balakrishnan J, Ciliz MK. *Adaptation and learning using multiple models, switching, and tuning*, IEEE Control Systems Magazine **15(3)** (1995),3751

[16] Colaneri P, Geromel JC. AstolfiA. *Stabilization of continuous-time switched nonlinear systems*, Systems and Control Letters, **57(1)** (2008), 95103

[17] Li J, Yang GH.,*Asynchronous fault detection filter design approach for discrete-time switched linear systems*, International Journal of Robust and Nonlinear Control, **70(1)** (2012), 409420

[18] Yang, Xuan, and Xiaoe Ruan. , *Iterative learning control for linear continuous-time switched systems with observation noise*, Transactions of the Institute of Measurement and Control, **41.4** (2019), No. 1-2, 1178-1185

[19] Ruan X, Bien Z and Wang Q., *Convergence properties of itera- tive learning control processes in the sense of the Lebesgue-p norm*, Asian Journal of Control, **14(4)** (2012), 10951107.

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